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High-order resonances of the quantum $\delta$-kicked accelerator

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Abstract – We report the observation of high-order resonances of the quantum $\delta$-kicked accelerator using a BEC kicked by a standing wave of light. The signature of these resonances is the existence of quantum accelerator modes. For the first time quantum accelerator modes were seen near 1/2, 2/3 and 1/3 of the half-Talbot time. Using a BEC enabled the internal momentum state structure of the modes and resonances to be studied for the first time. This structure has many similarities to that present in the fractional Talbot effect. We present a theory for this system based on rephasing of momentum orders and apply it to predict the behavior of the accelerator modes around a resonance of any order.

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Over the last decade the availability of ultra-cold atoms and the development of atom optical techniques has led to a revolution in the study of quantum chaotic systems. These techniques have allowed the quantum-chaotic dynamics of cold atoms subject to kicks from a standing wave of laser light to be observed in great detail. This system has been used in realizing the atom optics versions of the quantum $\delta$-kicked rotor (QDKR) [1] and the quantum $\delta$-kicked accelerator [2,3]. Other interesting classes of kicked systems studied theoretically include the kicked harmonic oscillator [4] and kicked Harper model [5].

One aspect of the behavior of these kicked systems that is of particular interest is the phenomenon of quantum resonance [6]. These resonances occur at distinct kicking periods and are characterized by a quadratic increase in the kinetic energy of atoms with certain initial momenta. To date most work has concentrated on the most easily observed of these resonances in the QDKR. These occur when the kicking period is an integer multiple of the half-Talbot time [7]. However, quantum resonances can also occur at kicking periods that are rational fractions $a/b$ (where $a$ and $b$ are integers) of the half-Talbot time [8]. These are what are known as “high-order” quantum resonances, with $b$ being the order of the resonance. Several of these resonances have been observed for the QDKR using both BEC [9] and a thermal sample of atoms [10]. High-order resonances have also been investigated theoretically for a $\delta$-kicked system within a harmonic potential (the $\delta$-kicked harmonic oscillator) [11]. Nevertheless, even though the signatures of a few high-order resonances have been detected, their rich internal structure has never been directly observed in any system.

In this letter we present the results of experiments using a Bose-Einstein condensate which show the structure and dynamics that gives rise to the quantum resonances. This is achieved through the first observation of quantum resonances of order 3, 4, and 6 in a QDKR which has been modified by the addition of a static linear potential created with the Earth’s gravity. Such a system is known as the quantum $\delta$-kicked accelerator (QDKA). The resonances can be described using two theoretical frameworks: one taking an approach where the quantum system is modeled using a classical-like mapping [12]; the other which is presented in this paper is based on the fractional Talbot effect [7] where the atom-light interaction is considered as diffraction at a phase grating. These complimentary pictures enable us to predict the ensemble behavior of the system and to explain the internal momentum state (diffraction order) structure of the quantum resonances observed in the experiments. Interference between these momentum states is also the reason that only atoms with certain initial momenta can participate in the resonance.

For the QDKA, the presence of the linear potential breaks the symmetry and allows a fixed momentum per kick to be imparted to a subset of atoms when the pulse period is close to a resonance time. Atoms which
are kicked in such a way as are said to be in a quantum accelerator mode (QAM) [3]. Fishman, Guarneri and Rebuzzini (henceforth referred to as FGR) developed a framework called $\epsilon$-classical theory [13], where a parameter $\epsilon$ proportional to the difference of the pulse period from one of the primary resonances plays the role of Planck’s constant. In the limit of pulse period very close to a primary resonance time, $\epsilon \to 0$, a classical mapping can be used to describe the system. This theory has been modified to develop a model for the QAM near high-order resonances [12]. The kicked accelerator can be described using a Hamiltonian

$$H = P^2/2M - Mg'x + h\phi_d \cos(Gx) \sum_n \delta(t' - nT),$$  \hspace{1cm} (1)

where the first term represents the kinetic energy ($P$ is the atomic momentum, $M$ is the atomic mass), the second term is a linear potential ($g'$ is an acceleration in the direction of the pulsed standing wave, $x$), and the third term is the $\delta$-kicked potential ($\phi_d$ is the kicking strength, $G = 2\pi/\lambda_G$, $\lambda_G$ is the spatial period of the potential, $T$ is the kicking period). One possible way to understand QAMs at high order resonances is to use the approach developed by FGR at the primary resonances where the single-kick evolution operator is written in a frame falling with the acceleration $g'$ as $U_\beta(t) = e^{-i\frac{\pi}{2}k\cos \theta}e^{-i\frac{\pi}{2}\hat{H}_\beta}$, where $\hat{H}_\beta = \frac{i}{l} \hat{J}^2 + \hat{I}((\pi l + \pi(\beta + \eta T + \eta/2))$ describes the effective Hamiltonian for free evolution between any two kicks separated by a time interval close to the resonance time $T_{1/2}$ (where $l$ is an integer and $T_{1/2} = 2\pi M/\hbar\beta$ is the half-Talbot time). The position and momentum co-ordinates are now represented by $\theta = Gx \bmod 2\pi$, and $\hat{I} = -ie\frac{\partial}{\partial \theta}$, respectively, and time represented by $\tau = 2\pi T/T_{1/2}$, with the parameter $\epsilon = 2\pi(T/T_{1/2} - l)$ measuring the closeness of the kick period to the primary resonance $T_{1/2}$. Note that $\epsilon$ appears in the place of Planck’s constant in the evolution operator and hence can formally be regarded as $\hbar$. Other variables include the quasimomentum $\beta$, the scaled kicking strength $\tilde{k} = \epsilon\phi_d$, and the scaled acceleration $\eta = Mg'T/hG).$ As with the kicked rotor, higher-order resonances are expected when $T$ is a rational fraction $(a/b)$ of $T_{1/2}$.

By analogy, for the high-order resonances $\epsilon$ can be defined as $\epsilon = 2\pi(T/T_{1/2} - a/b)$ and for small $|\epsilon|$, treating $\epsilon$ as Planck’s constant, the dynamics of the kicked accelerator near a higher-order resonance can be described using a mapping [12]

$$J_{t+1} = J_t + \tilde{k} \sin(\theta_{t+1}) + \eta \tau + \Delta S_t,$$

$$\theta_{t+1} = \theta_t + J_t,$$

where $J_t = I_t + \pi(\Delta \beta + \eta T + \eta/2) + 2\pi S_t/b$, and $\Delta S_t = 2\pi(S_{t+1} - S_t)/b$. The integers $S_t$ can take values from 1 to $b$ and are arbitrary. Thus there are $b^l$ maps for a given set of experimental parameters. For a periodic orbit with period $p$, jumping index $j$, satisfying equation

$$J_{(t+p)q} = J_{tq} + 2\pi j,$$

where $q$ is an integer satisfying equation $\Delta S_{t+q} = \Delta S_t$ [12], the average momentum is given by [12]

$$\bar{\rho}_{QAM} = -\lim_{T \to +\infty} \frac{\eta T}{\epsilon} + \left(\frac{2\pi i}{p} - \sum_{r=0}^{q-1} \Delta S_r\right) \frac{l}{q}. \hspace{1cm} (3)$$

The QAM with $j \neq 0$ are termed as higher-order QAM [14]. For the simple case of $\Delta S_r = 0$, and primary QAM ($j = 0$), the mean momentum of the periodic orbits is given by only the first term of eq. (3), identical to the ones at primary resonances. The mapping of this simple case is shown for two different higher-order resonances in fig. 3.

The general signature of a quantum resonance of the quantum $\delta$-kicked accelerator is expected to be the asymptotic divergence of a QAM’S momentum, $\bar{\rho}_{QAM}$, to infinity as $\epsilon \to 0$, that is, when the kicking period approaches the high-order resonance time. To experimentally observe these quantum resonances we subjected a BEC to pulses of standing wave light as described in detail in [2]. In brief, the BEC was created in an optical trap and consisted of approximately 30000 Rb-87 atoms in the $F = 1$, $5S_1/2$ level. After release from the trap, the BEC was kicked by 780 nm light which was 6.8 GHz detuned to the red of the atomic transition. For this detuning and $\phi_d \sim 2$ (the maximum used in the experiments) the spontaneous emissions per pulse for each atom is negligible at about $1.8 \times 10^{-3}$. This light propagated through two acousto-optic modulators (AOMs) to control the initial momentum and the acceleration of the atoms with respect to the standing wave. This was accomplished by driving the two AOMs with different frequencies. The kicking beam was oriented at $52^\circ$ to the vertical. Overall these parameters gave a value for the half-Talbot time of $T_{1/2} = 33.15 \mu s$. In order to vary the kicking strength $\phi_d$, the length of the kicking pulses was adjusted. Typically the pulse length was set at $1.8 \mu s$ giving $\phi_d \approx 1.5$. The finite pulse length does affect the dynamics of the system, particularly for states having momenta greater than $\sim 10\hbar G$. At our pulse lengths, such a state would move a significant fraction ($\sim 0.6$) of the standing wavelength. However, it is still possible to consider this system as a $\delta$-kicked rotor but with a reduced $\phi_d$ [15]. This is one of the factors which makes it difficult to observe high-order resonances since they require a large value of $\phi_d$ to become visible (see following discussion). The value of $\phi_d$ was estimated by comparing the relative population of various diffraction orders after one kick. Note that the population in the $n$-th state is given in terms of Bessel functions via $|J_n(\phi_d)|^2$ [16]. The momentum distribution of the BEC was measured by taking an absorption image $8 \text{ ms}$ after the completion of the kicking sequence. Finally it should be noted that the mean-field energy was weak enough that it could be ignored, making the Hamiltonian of the $\delta$-kicked accelerator a valid approximation.

Figure 1 shows experimental scans of the kicking period across two different high-order resonances. These figures were generated by horizontally stacking the absorption
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Fig. 1: Horizontally stacked momentum distributions for different kicking periods for (a) 40 kicks across \((1/2)\T_{1/2}\) \((a = 1\) and \(b = 2\)) for the effective acceleration \(g' = 6 \text{ms}^{-2}\), and \(\phi_d = 1.4\); (b) 30 kicks across \((1/3)\T_{1/2}\) \((a = 1\) and \(b = 3\)), \(g' = 4.5 \text{ms}^{-2}\) and \(\phi_d = 1.8\). The initial momentum was chosen such that the part of the mode below the resonance time is populated more strongly in the case of (a) and vice versa in the case of (b). The dashed curve is that of the \(\epsilon\)-classical theory in eq. (3).

Fig. 2: Horizontally stacked momentum distributions as a function of number of kicks \((t)\) for (a) \(T = 22.68 \mu s\), which is close to \((2/3)\T_{1/2}\), (b) \(T = 17.1 \mu s\), which is close to \((1/2)\T_{1/2}\), and (c) \(T = 72.4 \mu s\) which is close to \(2\T_{1/2}\). Note the different axes for (a), (b) and (c). The horizontal arrows in (a) and (b) show that primarily orders separated by \(b\hbar G\) participate in each of the QAMs. Dashed lines show the position of the QAM predicted by the \(\epsilon\)-classical theory of eq. (3). The end panels show the momentum distribution for 33 kicks (denoted by the downward arrow in the main panels). The value of \(g' = 6 \text{ms}^{-2}\) for these experiments. The location of the QAMs are shown by the vertical double arrows.

To investigate the properties of the QAM about the high-order resonances further, we conducted a series of experiments where kick number was increased at a fixed kicking period close to a resonance. Figure 2(a) is a scan of number of kicks close to \((2/3)\T_{1/2}\). This hints that the QAM consists primarily of momentum states separated by \(3\hbar G\) (indicated by horizontal arrows). The scan of kick number close to \((1/2)\T_{1/2}\) of fig. 2(b) shows much more clearly that the QAM is composed of momentum states separated by \(2\hbar G\). These momentum states are again emphasized by horizontal arrows. In contrast, at the Talbot time \((2\T_{1/2})\), the QAM includes neighboring momentum states as seen from fig. 2(c). The right hand side of each of these figures shows a plot of the population vs. momentum at 33 kicks (vertical arrows). Note again the separation of \(b\hbar G\) between the momentum states participating in a QAM. This behavior suggests that the QAM around the high-order resonances can form whenever the momentum orders separated by \(b\hbar G\) rephase during the time between the kicks. This is analogous to what has been postulated to occur (but never directly observed) for the kicked rotor resonances [9], and is consistent with what is known of the fractional Talbot effect [7].
which is close to \((2/3)\) of a momentum state participating in the QAM. Recall that the population of \((1/2)\) increases linearly with time (first term of eq. (3)), we that the momentum of states participating in a QAM kicks. Following the procedure of \([16]\), along with the result that the momentum of states participating in a QAM increases linearly with time (first term of eq. (3)), we obtain the condition on the initial momentum (in units of \(hG\)) at which a QAM appears as

\[
p_i = \frac{2\pi\nu}{\tau b} + b \frac{1}{\frac{\eta}{2b}},
\]

where \(\nu\) is an integer. It can be seen that the QAMs in the \(\delta\)-kicked accelerator are spaced in initial momentum by \(\Delta p_i = 2\pi/\tau b \approx 1/a\). This implies that the islands in the phase space map (which are attributed to the existence of the QAMs \([13]\)) shown in fig. 3 are separated in momentum by \(hG/a\). Figure 3 shows these phase space islands close to \((1/2)T_{1/2}\) in fig. 3(a) and \((2/3)T_{1/2}\) in fig. 3(b). The momentum axes in these maps cover a range of \(1hG\).

Thus to resolve the structures in phase-space within a unit cell and test the theory of eq. (4), it is necessary that the momentum width be much narrower than \(hG/a\). In our experiments the BEC had a momentum width of 0.056 \(hG\) \([17]\) which makes it an excellent candidate for this task. The size of the BEC in momentum is shown in fig. 3(a) with two dashed lines. In the \(\theta\) direction, BEC stretches across the whole phase space. Figure 4 shows the results from experiments in which the effective initial momentum of the BEC was changed by moving the standing wave using a difference in frequency between the kicking AOMs. This is equivalent to taking the strip shown between the dashed lines in fig. 3(a) and scanning it through the unit cell. The kicking period was near \((1/2)T_{1/2}\) in fig. 4(a) and \((2/3)T_{1/2}\) in fig. 4(b). Over a range of \(1hG\) of initial momentum, the QAM appears once at \((1/2)T_{1/2}\) and twice at \((2/3)T_{1/2}\). This demonstrates that the initial momenta at which the modes exist are

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**Fig. 3:** (Colour on-line) Phase space plot of the map (2) for (a) \(\phi_d = 1.5\), \(g' = 6\) ms\(^{-2}\) and \(T = 17.1\) \(\mu s\), which is close to \((1/2)\) \(T_{1/2}\), and (b) \(\phi_d = 1.8\), \(g' = 4.5\) ms\(^{-2}\) and \(T = 22.68\) \(\mu s\) which is close to \((2/3)\) \(T_{1/2}\). When a wave packet is trapped in an island, a QAM is formed \([19]\). The dashed lines in (a) show the width of the BEC.

**Fig. 4:** Horizontally stacked momentum distributions as a function of the initial momentum of the BEC before kicking, (a) 30 kicks with a period of 17.1 \(\mu s\) (close to \((1/2)\) \(T_{1/2}\), \(a = 1\)), and \(g' = 6\) ms\(^{-2}\), (b) 40 kicks with the period of 22.53 \(\mu s\) (close to \((2/3)\) \(T_{1/2}\), \(a = 2\)), and \(g' = 4.5\) ms\(^{-2}\). The position of the modes are indicated by the dashed lines. The panels (a1) and (b1) show the final momentum distribution for initial momenta at which a QAM does not appear. Panels (a2) and (b2) correspond to initial momenta at which a QAM is seen. The momentum range of QAMs is shown as double arrows in (a2) and (b2).
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separated by $\hbar G$ in the case of $(1/2)T_{1/2}$ ($a = 1$) and $\hbar G/2$ for $(2/3)T_{1/2}$ ($a = 2$) in agreement with eq. (4).

In conclusion, we have experimentally demonstrated the existence of high-order resonances in the quantum $\delta$-kicked accelerator. This was possible through the observation of QAMs near these resonances. The narrow momentum distribution of the BEC allowed us to observe the momentum state structure of the QAMs. It was found that QAMs near high-order resonances have a structure which is reminiscent of that produced by the fractional Talbot effect. Furthermore, we were able to explore the phase space structures predicted by $\epsilon$-classical theory of high-order resonances. This work opens the door towards the study of high-order QAMs [14] near high-order resonances. Other interesting questions include the effect of stronger mean-field interactions [18], the enhancement of QAM population using more complex initial states, and the effect of finite pulse length.

REFERENCES


