справедливо для сильно разогретой и одновременно сжатой системы.

Приведенные применения уравнения Боголюбова не исчерпывают все его возможности. Дальнейшее изучение этого уравнения - актуальная задача.

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NONLINEAR PARTIAL DIFFERENCE EQUATIONS FOR ISING MODEL n-POINT GREEN'S FUNCTIONS

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This contribution is concerned with recent developments and results for the n-point correlation functions of the two-dimensional Ising model. The major results to be discussed here have been reported briefly elsewhere, by McCoy, Wu, and the author[1].

The two-dimensional Ising model has already had a long history[2,3]. After the calculation of the partition function[4,5] and the spontaneous magnetization[6], great interest arose in the correlation functions[7,8,3]. There are several methods for obtaining those correlations. One class of methods is expressing them as determinants, often infinite block-Toeplitz determinants, which then have to be analysed further. Another class of methods is to derive identities between the correlations. In the first place there are linear identities[9-11], which can be generally formulated for Ising models on lattice Λ with arbitrary interactions and in arbitrary dimension as

$$\langle \sigma_A \rangle = (-1)^{|A\cap B|} \langle \sigma_A \exp\{ \sum_{C\subset\Lambda} [(-1)^{|B\cap C|}-1] K_C \sigma_C \} \rangle ,$$

$$-\beta \mathcal{H} \equiv \sum_{C\subset\Lambda} K_C \sigma_C , \quad \sigma_A \equiv \prod_{a\in A} \sigma_a . \tag{1}$$

The proof is rather easy after going to a quantum formulation in terms of the Pauli matrices at site a, $\sigma_a \equiv \sigma_a^x$, σ_a^y, σ_a^z. Then it follows that

$$\langle \sigma_A \rangle = \langle \sigma_A^x \rangle = \langle \sigma_B^y \sigma_B^y \sigma_A^x \rangle$$

$$= (-1)^{|A\cap B|} \langle \sigma_B^y \sigma_A^x \sigma_B^y \rangle \tag{2a}$$

and the result (1) follows from the cyclic property of trace (KMS-property) and

$$\sigma_B^y \exp\{\sum_C K_C \sigma_C^x\} = \exp\{\sum_C (-1)^{|B \cap C|} K_C \sigma_C^x\} \sigma_B^y . \tag{2b}$$

These identities express 2-point functions in 4-point functions, 4-point in 6-point, etc. Therefore, closure relations are needed to determine the full set of correlations from them, which is a well-known problem for the BBGKY hierarchy[12], although in one case this has been ingeneously circumvented[11]. There are also on planar lattices, nonlinear identities[13], which in general express n-point functions in terms of higher correlations, and related useful inequalities[14]. It is, however, also possible to get a closed set of quadratic identities involving n-point functions only[1], provided one includes order variables (spins) and disorder variables, which are related by the Kramers-Wannier duality[16]. We shall come back to this later.

The first real step towards the understanding of the Ising model correlations was the asymptotic expansion of the large distance behaviour of the two-point function[17]. Next the behaviour of particular n-point functions was studied[18,19], such as the four-spin correlation of spins on a line or of two pairs of neighbouring spins, energy-density correlations; the transverse correlations[20] could be obtained also. Then there were two major discoveries. In the first place, Kadanoff found that at the critical temperature the n-point functions could be expressed in a transparant way in terms of 2-point functions provided that all spins are well-separated[15,21]. This consequence of his hypothesis of reduction of critical fluctuations and algebra of local operators, has been checked by explicit calculations[19]. The second important discovery was for the two-point function in the so-called scaling limit, where distances are scaled with the correlation length upon going to the critical point. In 1973 Barouch, McCoy, Tracy, and Wu announced that it can be expressed in terms of the solution of a nonlinear differential equation and various consequences were given[22] before the details[23] appeared in 1976. These involve three steps. First the block-Toeplitz determinant is expressed in terms of one or two dispersion integral expansions, the n-th term being a 2n-or (2n-1)-fold integral. Then the scaling limit is taken and rotational invariance is checked. Finally the expansions are shown to be expressible in terms of the Painlevé function[24] of the third kind[25], using the results of the thesis of Myers[26]. The first two steps of this program have been carried out also for the n-point function[27]. In the papers mentioned above very explicit results have been given for the scaling functions and the corrections to scaling. Some recent work of Abraham[28] may be noted here also, where

based on previous work[29] a Wick theorem and a rigorous derivation of $\gamma = 7/4$ and $\delta = 15$ are given. The scaling limit results[27] for the n-point function have been used to study the string structure (collinearity versus noncollinearity)[30] and the effects of a small magnetic field[31]; furthermore, McCoy and Wu introduced a new kind of particles (imps)[32]. Critical point and scaling limit correlations were also studied by field theory[33] and S-matrix methods[34]. For more general models, the critical correlations at large separation were also inferred from field theory methods[35] and mappings to the Gaussian[36] and the Luttinger[37] model. Some of these treatments have been given for one-dimensional quantum models only, but by now there is a well-established relationship between those quantum chain systems and their two-dimensional classical counterparts[38,39].

Another milestone in the theory of Ising model n-point functions was the work of the Kyoto mathematicians Sato, Miwa, and Jimbo[40]. Using the Clifford algebra approach[5,7], they expressed the ratio of a certain matrix of auxiliary n-point order-disorder variable correlations and the n-point function itself as a dispersion integral expansion[23,27]. Then they studied the properties of this expansion in the scaling limit under analytic continuation of the parameters, (monodromy data) from which they concluded it to be an isomonodromy family with regular singularities. This in turn was enough for them to write down a system of linear differential equations. The integrability conditions (deformation theory) of this system then led to a completely integrable system of nonlinear differential equations, that for n=2 reduced to the Painlevé equation of the third kind[22,23]. Finally, they related the n-point function with the solution of the nonlinear system. In their treatment there are several nontrivial convergence problems which have been investigated in more detail by Palmer and Tracy[41]. Also, the work of Sato, Miwa, and Jimbo has stimulated a lot of further progress in the theory of differential equations and algebraic geometry[42] and statistical mechanics[43]. In particular, based on previous work on the one-dimensional XY-model in the (double) scaling limit[44] and on impenetrable bosons[45] they were able to write down Painlevé's differential equation of the fifth kind for the reduced density matrix of the boson problem. Independently of this, for the XY-model[46] and its subcase the Ising model in transverse field[47] other differential equations had been derived for the dynamic correlations at infinite temperature leading to predominantly Gaussian decay[48]. It is amusing to note here a very striking connection with recent work on U(N) gauge theory[69] which is made most explicit by looking at the Toeplitz determinant of modified Bessel functions

$$\det_{1\le i,j\le N} I_{i-j}(2\beta) = \exp\left\{\beta^2 - \sum_{n=1}^{\infty} \frac{1}{n} \sum_{k_1=0}^{\infty} \cdots \sum_{k_{2n}=0}^{\infty} \prod_{j=1}^{2n} J_{N+k_j+k_{j+1}+1}(2\beta)\right\}. \tag{3}$$

Here the correction terms to the Gaussian for finite N have been calculated using eqs. (11) and (12) of ref. 50, the series converging rapidly for real $\beta < (N+1)e^{-1}$; furthermore $\beta = iJt$ for the XY-model. Also a connection with chiral models is given[/51/]. Recently it has been stressed that knowledge of properties of the two-dimensional lattice models leads to further information on four-dimensional gauge theories[/52,53/]. Finally, nonperturbative field theory in 1+1 dimensions[/54/] has been done based on calculations for the correlation length[/55/] in the 8-vertex model and more results may be obtained using new results for the fermisystem[/56/] and the hard-hexagon model[/57/].

There is by now a list of exactly solvable models, for which the commutation relations[/39,58,59/] imply an infinite number of conservation laws. These models should be completely integrable in the sense that all correlations are to be obtained by, for example, the quantum inverse scattering method[/59/]. But so far beyond the Ising or XY-model, only the normalization[/60/] integral of the wave function has been constructed and some interesting results[/61/] in low order perturbation about the impenetrable boson system have been derived. However, for the Ising model and the one-dimensional XY-model there have been further developments for the Green's function on the lattice[/62-69/]. McCoy and Wu noted that the three step procedure for the Ising model two-point function in the scaling limit[/23/] could be generalized to the two-point function on the lattice[/62/], omitting the second step of taking the scaling limit and calculating differences involving the dispersion integral expansions for the third step. Another much more direct method has been given[/63/], which is also valid on general planar lattices. Then the uniqueness of the solution of the resulting set of partial difference equations was discussed[/64/]. Next results for n-point functions were announced[/65,66/]. Independently, Jimbo and Miwa[/67/] constructed difference equations for the two-point function on a diagonal of the lattice, and they also found a connection with the Painlevé equation of the sixth kind. Finally, new equations have been reported for the two-point function of the XY-model (transverse Ising chain)[/68,69/]. The new equations, when restricted to critical points, have the form of well-known completely integrable differential-difference (Toda-lattice)[/70/] or partial difference[/71/] equations.

One way of attacking the problem is using a newly formulated Wick theorem[/63,65,68,69/] which gives a set of equations for many other models also[/72-84/]. But more research is needed to construct a final equation for the free fermion model[/72/], the Ising model on a triangular lattice[/73/] or in field $i\pi/2kT$[/74/], Ising models with boundaries[/75/], periodic layering[/76-78/] or more general periodic structure[/79,80/], the Ising model with an interface[/81/] or a line of impurities[/82/], (see also Abe's related 1/n expansion work[/83/]), and last but not least the Ising model with quenched random layering[/84,78/], which gives rise to essential singularities[/85/] in contrast with the annealed case[/77,86/]. It should be noted that in view of the complete integrability many equations can be derived but that only a subset is needed to determine the problem, although it is not always easy to recognize the dependences[/65/].

The Wick theorem which will be used is[/63,68,69/]

$$\begin{aligned}
&\mathrm{Tr}\,(\Gamma_1 O_1 \Gamma_2 O_2 \Gamma_3 O_3 \Gamma_4 O_4)\ \mathrm{Tr}\,(O_1 O_2 O_3 O_4) \\
&= \mathrm{Tr}\,(\Gamma_1 O_1 \Gamma_2 O_2 O_3 O_4)\ \mathrm{Tr}\,(O_1 O_2 \Gamma_3 O_3 \Gamma_4 O_4) \\
&\quad - \mathrm{Tr}\,(\Gamma_1 O_1 O_2 \Gamma_3 O_3 O_4)\ \mathrm{Tr}\,(O_1 \Gamma_2 O_2 O_3 \Gamma_4 O_4) \\
&\quad + \mathrm{Tr}\,(\Gamma_1 O_1 O_2 O_3 \Gamma_4 O_4)\ \mathrm{Tr}\,(O_1 \Gamma_2 O_2 \Gamma_3 O_3 O_4),
\end{aligned} \tag{4}$$

where the Γ's are fermion operators satisfying the Clifford algebra anticommutation relations

$$\Gamma_k \Gamma_\ell + \Gamma_\ell \Gamma_k = 2\,\delta_{k\ell} \tag{5}$$

and the O's are products

$$O_p = \prod_\ell O_{p\ell}\ , \qquad (p = 1,2,3,4), \tag{6}$$

where the factors are either exponents of quadratic forms or linear combinations of Γ's, i.e. depending on p or ℓ

$$O_{p\ell} = \exp\Big\{\sum_{m,n} A^{p\ell}_{mn}\,\Gamma_m \Gamma_n\Big\} \tag{7a}$$

or

$$O_{p\ell} = \sum_k \lambda_{p\ell k}\,\Gamma_k\ . \tag{7b}$$

An alternative form is the theorem on compound Pfaffians:
Let $\{(i,j)\}$, for $i,j \in A\cup B$, be a triangular array, and let

$$\mathrm{Pf}\,(X) \equiv \mathop{\mathrm{Pf}}_{\{i,j\}\subset X} \{(i,j)\}\ , \tag{8a}$$

then

$$\frac{\mathrm{Pf}\,(A\cup B)}{\mathrm{Pf}(A)} = \mathop{\mathrm{Pf}}_{\{i,j\}\subset B}\left\{\frac{\mathrm{Pf}(A\cup\{i,j\})}{\mathrm{Pf}\,(A)}\right\}. \tag{8b}$$

Related results have been mentioned in the older mathematical literature[/87/]. The proof of the Wick theorem uses doubling of the Hilbert space and rotational invariance [/78,79/], as might have been expected for Gaussian integrals on Clifford algebra's[/88/]; combinatorial proofs of (8) can be given also.

The results will be given for the inhomogeneous Ising model on a rectangular lattice, with the interaction energy

$$-\beta\mathcal{H} = \sum_{M,N} \left(H_{MN}\sigma_{MN}\sigma_{M,N+1} + V_{MN}\sigma_{MN}\sigma_{M+1,N} \right). \tag{9}$$

This gives the Ising model on arbitrary planar lattices by setting appropriate bonds zero or infinity. The row-to-row transfer matrices are defined in the usual way as

$$T_M = \exp\left\{ \sum_N H_{MN}\sigma_N^x\sigma_{N+1}^x \right\}, \tag{10a}$$

$$T_{M+\frac{1}{2}} = \left[\prod_N \{2\sinh(2V_{MN})\}^{1/2} \right] \exp\left\{ \sum_N V_{MN}^* \sigma_N^z \right\}, \tag{10b}$$

where

$$\sinh(2V_{MN}^*)\sinh(2V_{MN}) = \cosh(2V_{MN}^*)\tanh(2V_{MN}) = 1, \tag{11}$$

and the Pauli matrices are chosen in the representation

$$\sigma^x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}. \tag{12}$$

Eqs. (10a) and (10b) give the Boltzmann weights of the horizontal interactions in row M and the vertical interactions between rows M and M+1, respectively. More precisely, $T_{M+\frac{1}{2}}$ is a matrix with elements

$$T_{\{\sigma_{MN}\}|\{\sigma_{M+1,N}\}} = \prod_N e^{V_{MN}\sigma_{MN}\sigma_{M+1,N}}, \tag{13a}$$

or

$$\begin{aligned} T_{M+\frac{1}{2}} &= \bigotimes_N \begin{pmatrix} e^{V_{MN}} & e^{-V_{MN}} \\ e^{-V_{MN}} & e^{V_{MN}} \end{pmatrix} \\ &= \bigotimes_N \left(e^{V_{MN}} + \sigma^z e^{-V_{MN}} \right) \\ &= \bigotimes_N \left[\{2\sinh(2V_{MN})\}^{1/2} \exp(V_{MN}^*\sigma^z) \right], \end{aligned} \tag{13b}$$

giving (10b).

In the following, we let M,N,K,L denote integers and m,n,k,ℓ denote integers and half-integers. Also we define a "time-evolution"

$$A(m) \equiv \left(\prod_{k\le m} T_k \right) A \left(\prod_{k\le m} T_k \right)^{-1} \tag{14}$$

and a "time-ordering" or M-ordering operator $\mathcal{T}$

$$\mathcal{T}\left[A(m_1) \dots A(m_s) \right] = A(m_{P1}) \dots A(m_{Ps}), \tag{15a}$$

where the permutation P is such that

$$m_{P1} \le m_{P2} \le \dots \le m_{Ps}. \tag{15b}$$

Then the spin correlations can be rewritten

$$\langle \sigma_{M_1N_1} \dots \sigma_{M_sN_s} \rangle = \langle \mathcal{T}[\sigma_{N_1}^x(M_1) \dots \sigma_{N_s}^x(M_s)] \rangle, \tag{16}$$

where the canonical average now reads

$$\langle \dots \rangle = \frac{\mathrm{Tr}\left(\dots \prod_m T_m \right)}{\mathrm{Tr}\left(\prod_m T_m \right)}. \tag{17}$$

We also have to take account of correlations involving the disorder variables of Kadanoff and Ceva [15]. Such correlations are expressed as $\mathrm{Tr}(e^{-\beta\mathcal{H}'})/\mathrm{Tr}(e^{-\beta\mathcal{H}})$, where in $-\beta\mathcal{H}'$ the whole string of vertical interactions to the right of the disorder variable μ_{MN}, located at $(M+\frac{1}{2}, N+\frac{1}{2})$ of the lattice, is replaced by minus itself, i.e., $V_{MK} \to -V_{MK}$, $(K>N)$, see Fig. 1. In the corresponding transfermatrix $T'_{M+\frac{1}{2}}$, we then

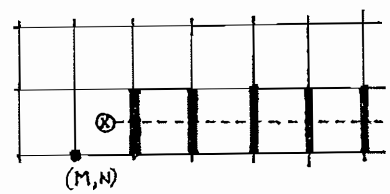

Fig. 1

have the replacement

$$\begin{aligned} \begin{pmatrix} e^{V_{MK}} & e^{-V_{MK}} \\ e^{-V_{MK}} & e^{V_{MK}} \end{pmatrix} &\longrightarrow \begin{pmatrix} e^{-V_{MK}} & e^{V_{MK}} \\ e^{V_{MK}} & e^{-V_{MK}} \end{pmatrix} = \\ &= \sigma^z \begin{pmatrix} e^{V_{MK}} & e^{-V_{MK}} \\ e^{-V_{MK}} & e^{V_{MK}} \end{pmatrix} \\ &= \sigma^z \left(e^{V_{MK}} + \sigma^z e^{-V_{MK}} \right), \end{aligned} \tag{18}$$

so that, cf. eqs. (10), (13),

$$T'_{M+\frac{1}{2}} = \Big(\prod_{K>N} \sigma^z_K\Big)\, T_{M+\frac{1}{2}}\,. \tag{19}$$

Therefore the disorder variable μ_{MN} is represented by

$$\mu^x_N(M) \equiv \prod_{K>N} \sigma^z_K(M) = \mu^x_N(M+\tfrac{1}{2}). \tag{20}$$

It is now convenient to introduce the redefinitions

$$\begin{aligned} \mu_{MN} &\equiv \sigma_{M+\frac{1}{2},N+\frac{1}{2}}\,, \\ \mu^x_N(M) &\equiv \sigma^x_{N+\frac{1}{2}}(M+\tfrac{1}{2})\,. \end{aligned} \tag{21}$$

Then a correlation involving both order variables (spins) and disorder variables is rewritten as, cf. (16),

$$\langle \sigma_{m_1 n_1} \cdots \sigma_{m_s n_s} \rangle = \langle \mathcal{T}[\sigma^x_{n_1}(m_1)\cdots\sigma^x_{n_s}(m_s)] \rangle, \tag{22}$$

where m_j, n_j are always both integer for order variables or both half-integer for disorder variables.

We now define Clifford algebra operators by

$$\begin{aligned} \Gamma_N &\equiv \sigma^x_N \mu^x_N = \sigma^x_N \prod_{K>N} \sigma^z_K\,, \\ \Gamma_{N-\frac{1}{2}} &\equiv i\,\sigma^x_N \mu^x_{N-1} = \sigma^y_N \prod_{K>N} \sigma^z_K\,, \end{aligned} \tag{23}$$

from which one has (5),

$$\begin{aligned} \sigma^x_N &= \Gamma_N \prod_{K>N} (i\,\Gamma_{K-\frac{1}{2}} \Gamma_K)\,, \\ \sigma^y_N &= \Gamma_{N-\frac{1}{2}} \prod_{K>N} (i\,\Gamma_{K-\frac{1}{2}} \Gamma_K)\,, \\ \sigma^z_N &= i\,\Gamma_{N-\frac{1}{2}} \Gamma_N\,, \end{aligned} \tag{24}$$

$$\begin{aligned} \mu^x_N &= \prod_{K>N} (i\,\Gamma_{K-\frac{1}{2}} \Gamma_K)\,, \\ \mu^z_N &= \sigma^x_N \sigma^x_{N+1} = i\,\Gamma_N \Gamma_{N+\frac{1}{2}}\,, \end{aligned} \tag{25}$$

and

$$\begin{aligned} T_M &= \exp\Big[\, i \sum_N H_{MN}\, \Gamma_N \Gamma_{N+\frac{1}{2}} \Big]\,, \\ T_{M+\frac{1}{2}} &= \Big[\prod_N \{2\sinh(2V_{MN})\}^{1/2}\Big] \exp\Big[\, i \sum_N V^*_{MN}\, \Gamma_{N-\frac{1}{2}} \Gamma_N \Big]\,. \end{aligned} \tag{26}$$

The transfer matrices (26) can be given by one equation only using the notation

$$V^*_{MN} \equiv H_{M+\frac{1}{2},N-\frac{1}{2}}\,, \tag{27}$$

which expresses the idea that a vertical interaction between (M,N) and (M+1,N) on the lattice becomes a horizontal interaction between $(M+\frac{1}{2},N-\frac{1}{2})$ and $(M+\frac{1}{2},N-\frac{1}{2})$ on the dual lattice, and using the fact that the constant factor in (10b) or (26) cancels out between the denominator and the numerator of (17). Therefore we can write

$$T_m = \exp\Big\{\, i \sum_{\substack{n \\ m+n=\text{integer}}} H_{mn}\, \Gamma_n \Gamma_{n+\frac{1}{2}} \Big\}\,. \tag{28}$$

The Clifford algebra operators are important since they obey a linear "time-evolution",

$$\begin{aligned} \Gamma_n(m) &= \cosh(2H_{mn})\,\Gamma_n(m-\tfrac{1}{2}) - i\sinh(2H_{mn})\,\Gamma_{n+\frac{1}{2}}(m-\tfrac{1}{2})\,, \\ \Gamma_{n+\frac{1}{2}}(m) &= \cosh(2H_{mn})\,\Gamma_{n+\frac{1}{2}}(m-\tfrac{1}{2}) + i\sinh(2H_{mn})\,\Gamma_n(m-\tfrac{1}{2})\,, \end{aligned} \tag{29}$$

for m+n = integer, as can be seen calculating expressions $T_m \Gamma_n T_m^{-1}$.

We have now constructed a very symmetric formalism treating lattice and dual lattice on completely equal footing. The last subtle difference left, the powers of i in (24), (25), can be eliminated using the redefinition

$$\sigma_n \equiv \prod_{k \geq n} \Gamma_k\,, \tag{30}$$

so that

$$\begin{aligned} \sigma^x_N &= (-i)^N \sigma_N\,, \\ \mu^x_N &= (-i)^N \sigma_{N+\frac{1}{2}}\,. \end{aligned} \tag{31}$$

One then has, for m+n = integer,

$$\sigma_n T_m = T_m \sigma_n \tag{32}$$

and

$$\begin{aligned} \Gamma_n(m)\,\sigma_n(m) &= \sigma_{n+\frac{1}{2}}(m) = \sigma_{n+\frac{1}{2}}(m+\tfrac{1}{2})\,, \\ \Gamma_{n-\frac{1}{2}}(m)\,\sigma_n(m) &= \sigma_{n-\frac{1}{2}}(m) = \sigma_{n-\frac{1}{2}}(m+\tfrac{1}{2})\,, \\ \Gamma_n(m-\tfrac{1}{2})\,\sigma_n(m) &= \Gamma_n(m-\tfrac{1}{2})\,\sigma_n(m-\tfrac{1}{2}) = \sigma_{n+\frac{1}{2}}(m-\tfrac{1}{2})\,, \\ \Gamma_{n-\frac{1}{2}}(m-\tfrac{1}{2})\,\sigma_n(m) &= \Gamma_{n-\frac{1}{2}}(m-\tfrac{1}{2})\,\sigma_n(m-\tfrac{1}{2}) = \sigma_{n-\frac{1}{2}}(m-\tfrac{1}{2})\,. \end{aligned} \tag{33}$$

The equations (33) are summarized in fig.2. The order variables $\sigma_N(M)$ are located at integer lattice coordinates; the disorder variables $\sigma_{N+\frac{1}{2}}(M+\frac{1}{2})$, at half-integer coordinates, both indicated by black dots. The spinor variables $\Gamma_n(m)$ are located at the lattice positions $(m+\frac{1}{2},n+\frac{1}{2})$, represented by small circles. A neighbouring order- and disorder variable pair is related by multiplying in front with the spinor variable corresponding to the small circle halfway between the two dots.

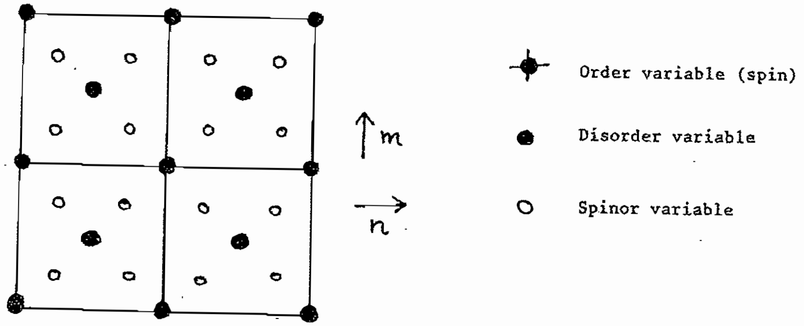

Fig.2.

We are now in a position to use the Wick theorem (4), noting that the order- and disorder variables (30), (31), the spinor variables $\Gamma_n(m)$ of (23), (14), and the transfer matrices (28) are all of the form prescribed by (6) and (7), and can be considered as contributions to the "scalar" factors O_1, O_2, O_3, and O_4. So we only have to designate four factors Γ_1, Γ_2, Γ_3, Γ_4 in fig.2. This leads to a large number of quadratic identities between correlations with order- and disorder variables, using (29) and (33) where necessary. There is really only one difficulty left. In the identification of the contributions in the Wick theorem with correlations (16), (22), the "time-ordering" operator $\mathcal{T}$ has to be taken into account. As a consequence, the identities may have to be amended if certain coordinates coincide or are on the same horizontal line, whenever $\mathcal{T}$ acts differently on the different contributions to the identity in question. In the first place there are exceptional points (source terms, boundary conditions) for equal positions, where the n-point functions are connected with the (n-2)-point functions, see also eq. (11) of ref. 63 for the n=2 case. In the second case there are half-line violations in terms of minus signs[15,1] which can be understood in a very simple picture of branch points, branch cuts, and two-sheetedness on the lattice, whenever a disorder variable moves around an order variable. This can be seen from (30) or

$$\sigma_n(m) = \sigma_n(m-\tfrac{1}{2}) = \prod_{k\geq n} \Gamma_k(m) = \prod_{k\geq n} \Gamma_k(m-\tfrac{1}{2}), \qquad (34)$$

with m+n = integer, from which

$$[\,\sigma_n(m)\,,\,\sigma_{n'}(m)\,]_- = 0 \quad , \quad n+n' = \text{integer}, \qquad (35)$$

but

$$\begin{aligned} &[\,\sigma_n(m)\,,\,\sigma_{n'}(m\pm\tfrac{1}{2})]_- = 0 \quad , \quad n<n', \\ &[\,\sigma_n(m)\,,\,\sigma_{n'}(m\pm\tfrac{1}{2})]_+ = 0 \quad , \quad n>n', \end{aligned} \qquad (36)$$

for m,n integer and n' half-integer.

It is now time to write down the main results, which are expressed in the notations

$$\sigma\,[M_1, N_1\,; \dots ;\, M_n, N_n] \equiv \langle \sigma_{M_1N_1}\sigma_{M_2N_2} \cdots \sigma_{M_nN_n}\rangle\,, \qquad (37)$$

and

$$\mu_{ij}\,[M_1, N_1\,; \dots ;\, M_n, N_n] \equiv \langle \sigma_{M_1N_1} \cdots \mu_{M_iN_i} \cdots \mu_{M_jN_j} \cdots \sigma_{M_nN_n}\rangle, \qquad (38)$$

with disorder variables on sites (M_i,N_i) and (M_j,N_j). In the following the arguments of σ and μ_{ij} will not be given except as far as needed, and μ_{ij} is defined to be antisymmetric in i and j. Correlations involving more disorder variables are expressed as Pfaffians in terms of the correlations with two disorder variables, i.e., from the Wick theorem it follows that

$$\sigma\,\mu_{ijkl} = \mu_{ij}\mu_{kl} - \mu_{ik}\mu_{jl} + \mu_{il}\mu_{jk}\,. \qquad (39)$$

Furthermore,

$$\begin{aligned} S_1(M,N) &\equiv \sinh(2H_{MN})\,, \quad C_1(M,N) \equiv \cosh(2H_{MN})\,, \\ S_2(M,N) &\equiv \sinh(2V_{MN})\,, \quad C_2(M,N) \equiv \cosh(2V_{MN})\,. \end{aligned} \qquad (40)$$

Then the results are, without spelling out the details connected with the point- and half-line violations mentioned before here,

$$\begin{aligned} &S_1(M_i,N_i)S_2(M_j,N_j)\{\sigma\sigma[M_i,N_i+1;M_j+1,N_j] - \sigma[M_i,N_i+1]\,\sigma[M_j+1,N_j]\} \\ &= \mu_{ij}\,\mu_{ij}[M_i-1,N_i;M_j,N_j-1] - \mu_{ij}[M_i-1,N_i]\,\mu_{ij}[M_j,N_j-1]\,, \end{aligned} \qquad (41)$$

for $i\neq j$,

$$\begin{aligned} &S_1(M_i,N_i)S_1(M_j,N_j)\{\sigma\sigma[M_i,N_i+1;M_j,N_j+1] - \sigma[M_i,N_i+1]\,\sigma[M_j,N_j+1]\} \\ &= \mu_{ij}[M_i-1,N_i]\,\mu_{ij}[M_j-1,N_j] - \mu_{ij}\,\mu_{ij}[M_i-1,N_i;M_j-1,N_j]\,, \end{aligned} \qquad (42a)$$

$$\begin{aligned} &S_2(M_i,N_i)S_2(M_j,N_j)\{\sigma\sigma[M_i+1,N_i;M_j+1,N_j] - \sigma[M_i+1,N_i]\,\sigma[M_j+1,N_j]\} \\ &= \mu_{ij}[M_i,N_i-1]\,\mu_{ij}[M_j,N_j-1] - \mu_{ij}\,\mu_{ij}[M_i,N_i-1;M_j,N_j-1]\,, \end{aligned} \qquad (42b)$$

for $i\neq j$ and $(M_i,N_i)\neq(M_j,N_j)$,

$$S_1(M_\ell,N_\ell)\{\sigma\mu_{ij}[M_\ell,N_\ell+1]-\sigma[M_\ell,N_\ell+1]\mu_{ij}\}$$
$$=\mu_{\ell i}[M_\ell-1,N_\ell]\mu_{\ell j}-\mu_{\ell i}\mu_{\ell j}[M_\ell-1,N_\ell]\,, \tag{43a}$$

$$S_2(M_\ell,N_\ell)\{\sigma\mu_{ij}[M_\ell+1,N_\ell]-\sigma[M_\ell+1,N_\ell]\mu_{ij}\}$$
$$=\mu_{\ell i}\mu_{\ell j}[M_\ell,N_\ell-1]-\mu_{\ell i}[M_\ell,N_\ell-1]\mu_{\ell j}\,, \tag{43b}$$

for $\ell \neq i \neq j$. There are many more equations[1], which can be obtained by interchanging order- and disorder variables or replacing in some of the disorder variables M by M-1 and/or N by N-1. These equations though linearly independent, are consequences of (41), (42), and (43), i.e., they are algebraically dependent.

In the translationally invariant case,

$$H_{MN} \equiv H\,, \quad V_{MN} \equiv V\,, \tag{44}$$

there is one more equation which can be derived using Wick theorems and translation invariance several times:

$$S_1\{\sigma[M_i,N_i+1]\mu_{ij}[M_i,N_i-1]+\sigma[M_j,N_j+1]\mu_{ij}[M_j,N_j-1]\}$$
$$+S_2\{\sigma[M_i+1,N_i]\mu_{ij}[M_i-1,N_i]+\sigma[M_j+1,N_j]\mu_{ij}[M_j-1,N_j]\}$$
$$+\sum_{\ell\neq i,j}\{\mu_{\ell i}[M_\ell,N_\ell-1]\mu_{\ell j}[M_\ell-1,N_\ell]-\mu_{\ell i}[M_\ell-1,N_\ell]\mu_{\ell j}[M_\ell,N_\ell-1]\}$$
$$=2C_1C_2\,\sigma\mu_{ij}\,. \tag{45}$$

It goes to far to sketch the derivation here. For n=2 this equation is a first (sum) integral of (41) and (42), see ref.[63]. Also in the scaling limit an integral of eqs. (41), (42), (43) can be written down[1], which comes from precisely half of eq. (45), the sum vanishing in the scaling limit. Finally, using the above equations and rotational invariance, the equations of Sato, Miwa, and Jimbo[40] for the n-point functions in the scaling limit, can be recovered. An important future problem is trying to find analogues of eq. (45) for more general Ising models[72-84] or to search for identities for Green's functions in more general systems with a duality transformation and with infinitely many conservation laws.

It is a pleasure to thank Professor B.M. McCoy and Professor T.T. Wu, who greatly contributed to the research presented here. This work is supported in part by the National Science Foundation under Grant No. PHY-79-06376A01.

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ISING MODEL AND DIMER COVERING ON LATTICES EMBEDDED IN MANIFOLDS WITH GENUS > 1

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In statistical mechanics the studies of critical phenomena and phase transitions have long been, to a large extent, centered around the lattice systems. Besides the intrinsic interest connected to the lattice systems as models of real physical situations, one is in general attracted to their consideration by the possibility of obtaining exact non-trivial solutions.

For a reason of that sort space-time lattices were recently introduced as a technical device to obtain cut-off field theories, so that their solutions would give some insight in field theories defined in continuum Minkowski space-time. Once a lattice field theory has been formulated, the original problem becomes one of statistical mechanics. The general philosophy is that at the critical point the theory should lose memory of the lattice structure, and the continuous space-time symmetries be recovered. Lattice gauge theories are especially remarkable for their relation to the classical spin systems of statistical mechanics/1/.

Yet the soluble problems are very few in number, and the Ising model still stands at the very frontiers of present knowledge. Its only drawback: so far no exact solution has been found in more than two dimensions/2/.

On the other hand a great deal of interest has recently risen in the so-called glassy states of solids. The latter are amorphous systems exhibiting a typical long range positional disorder of atoms. This results from the long range random deviation from the perfect form of densest lattice packing of a set of hard spheres. Typically such systems show a local (short range) order: five binding rings originated from the prevalence of close-packed ordered sets of 5 tetrahedra/3/.

Tetrahedra do not fill $\mathbb{R}^3$ regularly but – at the expense of small elastic