

# Supplement on Special and General Relativity

January 2020

by P. Shull

## Special Relativity Introduction

Robert Hooke (1667, a professor at Gresham) caused light to form interference patterns, implying light is wavelike and needs a medium to travel through.

Astronomer and mayor Olaf Roemer (1675), observing regular variations in the predicted eclipse times for Jupiter's moons, found that the speed of light ( $c$ ) is finite:  $2 \times 10^5$  km/s (really  $3 \times 10^5$  km/s — light circles Earth 7.5 times in 1 s).

Newton thought  $c$  was infinite, with all observers agreeing on absolute lengths and times.

Albert Michelson and Edward Morley (1887, professors at Case Western Reserve) found Earth's velocity did not affect the speed of light beams, meaning there is no medium!

Albert Einstein developed Special Relativity (1905) for objects in uniform relative motion, using two assumptions:

#1:  $c$  in empty space is same for all observers, *regardless of their relative motion*.

#2: the laws of physics are the same for all observers, *regardless of their relative motion*.

## Some Consequences of Special Relativity

Relativistic effects are obvious only above about  $0.5c$ , and Einstein's equations become Newtonian for speeds  $v \ll c$ ! The Lorentz factor  $\gamma$  (gamma) appears in many equations:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

$\gamma$  is always positive, equals 1 when  $v = 0$ , and becomes infinite as  $v$  approaches  $c$ . At half the speed of light,  $\gamma = 1.15$ .

(1) The distance interval  $\Delta x$  and time interval  $\Delta t$  between two events as measured by different observers in relative motion are different (!) because space and time are interlinked. However, the square of the spacetime interval  $\Delta s$  between the two events is the same for all observers:

$$(\Delta s)^2 = c^2 (\Delta t)^2 - (\Delta x)^2$$

(2) The sum of two relative velocities  $u$  and  $v$  is always  $\leq c$ ! Example: if a conveyor belt moves at speed  $u$  relative to the ground, and you are walking on the belt at speed  $v$  relative to the belt, then a person standing on the ground would measure your speed relative to the ground,  $w$ , to be

$$w = \frac{u + v}{1 + (uv / c^2)}$$

(3) Clocks moving away or sideways relative to you appear to run slowly, but faster if approaching. Let  $\Delta t$  be the time interval between successive "ticks" for a clock at rest relative to you. Let  $\Delta t_{obs}$  be the time interval you measure between those ticks when the clock is moving relative to you. Let "across" refer to motion perpendicular to your line of sight to the clock. Let "along" refer to motion parallel to your sight line. Then

$$\Delta t_{obs} = \gamma \Delta t \quad (\text{across})$$

$$\Delta t_{obs} = \gamma (1 \pm v/c) \Delta t \quad (\text{along, } -v \text{ if approaching})$$

Twins Paradox: Consider identical twins on planet Earth. If one departs on a fast spaceship, turns around, and returns, the traveler will be younger than the twin who stayed behind!

(4) Light sources moving away or sideways relative to you appear redder, but bluer if approaching. Let  $\lambda$  (lambda) be the wavelength of a light source at rest. The longer the wavelength, the redder the light. Then

$$\lambda_{obs} = \gamma \lambda \quad (\text{across})$$

$$\lambda_{obs} = \gamma (1 \pm v/c) \lambda \quad (\text{along, } -v \text{ if approaching})$$

(5) Moving objects of length  $L$  (when at rest) appear rotated, therefore shorter:

$$L_{obs} = L / \gamma \quad (\text{across})$$

(6) The kinetic energy  $E_{kin}$  required to accelerate any mass  $m$  becomes infinite as its speed  $v$  approaches  $c$ :

$$E_{kin} = (\gamma - 1) mc^2$$

(7) Matter and energy can be transformed into each other. A mass  $m$ , even at rest, has a rest energy  $E$ :

$$E = mc^2 \quad (\text{1 pound of matter possesses explosive energy potential of 10 MT of TNT!})$$

### General Relativity Introduction

Published in 1916. At Zurich, physics professor Einstein collaborated with math professor Marcel Grossman, who introduced him to the equations of curved space developed by math professor Georg Riemann in Göttingen in 1854.

In General Relativity, Special Relativity's restriction is lifted: acceleration is permitted.

It is also based on the Principle of Equivalence (1908): Acceleration and gravity are locally indistinguishable. Think of how your weight feels in an elevator that is just beginning to rise, or just beginning to sink.

### Some Consequences of General Relativity

One could say that space and time are affected by the presence of mass and energy.

(1) Spacetime is curved (non-Euclidean) because the squared spacetime interval  $(\Delta s)^2$  is more mathematically complex than in flat (Euclidean) spacetime.

(2) Gravity is interpreted as the curving of spacetime by matter and energy. Equivalent concentrations of matter and energy produce identical effects. Effects for a body of mass  $M$  and radius  $R$  become apparent when the gravitational Lorentz factor  $\gamma$  is noticeably larger than 1:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{M/M_{Sun}}{R/3 \text{ km}}\right)^2}}$$

(3) Trajectories of both matter and light are curved by gravity.

(4) Time (clocks etc.) slows down in gravitational fields or under acceleration:

$$\Delta t_{obs} = \gamma \Delta t$$

(5) Light will be redshifted while ascending out of a gravitational field, and blueshifted while entering one.

(6) Accelerating masses produce gravity waves, which are ripples moving at light speed in the objects' normal gravitational field patterns. Example: black holes closely orbiting each other cause periodic distortions in the physical shapes of objects elsewhere!